

The Effect of Cosmological Background Dynamics on the Spherical Collapse in MOND

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Abstract

The effect of background dynamics of the universe on formation of large scale structures in the framework of Modified Newtonian Dynamics (MOND) is investigated. A spherical collapse model is used for modeling the formation of the structures. This study is done in two extreme cases: (i) assuming a universe with a low-density baryonic matter without any cold dark matter and dark energy; (ii) a dark energy dominated universe with baryonic matter, without cold dark matter. We show that for the case (ii) the structures virialize at lower redshifts with larger radii compared to the low-density background universe. The dark energy slow downs the collapse of the structures. We show that our results are compatible with recent simulations of the structure formation in MOND.

Keywords: gravitation-galaxies: formation-cosmology: theory-dark matter-large scale structure of universe

1. Introduction

Asymptotic flat rotation curves of spiral galaxies reveal the missing mass problem in galactic scales (Bosma , 1981; van Albada et al. , 1985). This problem can be solved by cold dark matter (CDM) hypothesis, or alternative theories of gravitation (Carroll et al. , 2004; Halle et al. , 2008; Sobouti et al. , 2009; Zhao , 2007). The CDM hypothesis as a dominant paradigm together with the cosmological constant makes the standard model of cosmology (Spergel et al. , 2006). Despite many efforts for direct detection of dark matter, it has not been observed in non-gravitational experiments. On the other hand, results from high resolution N-body simulations do not seem to be compatible with the observations on the galactic scales and also provide incompatible spatial distribution of the sub-halos (Moore et al. , 1999; Klypin et al. , 1999; Metz et al. , 2008).

The Modified Newtonian Dynamics (MOND) paradigm is one of the alternative theories that was proposed by Milgrom (1983) to explain the dynamics of gravitational system. In MOND, the Newton's second law is modified as $\mu(a/a_0)\mathbf{a} = \mathbf{a}_N + \nabla \times \mathbf{H}$, where \mathbf{a}_N is the Newtonian acceleration vector, \mathbf{a} is the MONDian acceleration vector, $a = |\mathbf{a}|$ is the absolute value of MONDian acceleration, μ is an interpolating function which is used for transition from the Newtonian to the MOND regime, and $a_0 \simeq 1.2 \times 10^{-10} \text{ ms}^{-2}$ is the MONDian threshold acceleration (Bekenstein & Milgrom , 1984). The value of the curl field \mathbf{H} depends on the boundary conditions and the spatial mass distribution, and vanishes only for some special symmetries (Bekenstein & Milgrom , 1984). This modification successfully explains the flat rotation curves of spiral galaxies at the large distances. Below a characteristic acceleration a_0 , $\mu(x) \simeq x$ and in the absence of curl field H , the Newton's second law is approximated as $a = \sqrt{a_0 a_N}$, the so called deep MOND regime. MOND has recently been generalized to a general-relativistic version which is called TeVeS (Bekenstein , 2004).

One of the challenging issues for any gravitational theory is to test its ability to reproduce the large scale structure formation in the universe. In MOND, the large scale structure formation has been studied by Sanders (1998) in a low-density universe, i.e. the universe without dark energy and dark matter. He showed that in MONDian cosmology, a patch of the universe smaller than the horizon size evolves with a different rate than the background. Hence the structures naturally can be formed through the scale dependent dynamics. In this scenario small structures form faster than larger ones (bottom-up

hierarchical model). Note that in this model, every point in the space can be assumed as a center of collapse and the initial fluctuation of baryonic matter has no significant role in the formation of structures. In order to solve this problem, the association of MOND force can be considered as a peculiar acceleration in a finite-size region and the structure is formed from the initial baryonic fluctuation (Sanders , 2001). It has been shown that, this assumption leads to rapid growth of structures and the power spectrum of the structure is similar to that of obtained from Λ CDM universe (Sanders , 2001).

In the scale of clusters of galaxy, MOND reduces the discrepancy between dynamical and detectable mass to a factor of 2-3 in the scales of clusters of galaxy, but does not remove it completely (The & White , 1988; Sanders , 2003; Pointecouteau & Silk , 2005). Sanders (2003) speculated on the remaining missing mass and showed that 2-eV neutrinos which can aggregate on the scales of clusters of galaxy are a possible candidate to interpret the remaining missing mass (Sanders , 2003).

Up to present day, several attempts in the cosmological structure formation have been done by means of MONDian N-body simulation (Nusser , 2002; Knebe et al. , 2004; Llinares et al. , 2008).

Malekjani, Rahvar and Haghi (2009, here after MRH09) showed that in MOND, a uniform distribution of matter finally virializes with a power law profile in which the virialization process takes place gradually from the center of structure to the outer parts (Malekjani et al. , 2009a). In this work, we discuss the effect of *background dynamics* on the growth rate of the structures. In the case of universe dominated by cosmological constant, the spherical collapse is similar to that in CDM model, except the cosmological constant that changes the growth of the structure though altering the background dynamics. The main goal of the paper is to understand the effect of background dynamics of the universe on the evolution of spherical over-dense structures in the framework of MOND. It is possible to achieve this goal even if we consider only the pure baryonic matter as a single component of matter content of the universe and avoid the complexity of neutrino problem in the scales of clusters of galaxies. Using the simple baryonic spherical collapse model in the context of MOND, we study the effect of generic variable dark energy background model on the scenario of structure formation.

The paper is organized as follows: In section 2 we introduce the spherical collapse in MOND. In section 3 we calculate the effect of background dynamics on the structure formation in MOND. The paper is concluded in section 4.

2. Spherical collapse in MOND

The following is a brief review of MONDian cosmology and the spherical collapse in the framework of MOND (Sanders , 1998). In the scales smaller than Hubble radius, the dynamics of the universe is derived from the Hubble equation as

$$H^2(a) = H_0^2[\Omega_b^0 a^{-3} + \Omega_r^0 a^{-4} - (\Omega_t^0 - 1)a^{-2} + \Omega_{de}], \quad (1)$$

where H_0 is the Hubble parameter at the present time, Ω_b^0 , Ω_r^0 and Ω_t^0 are the baryonic matter, radiation and total energy density parameters, respectively, and Ω_{de} is the density parameter of dark energy. In MONDian cosmology, one can consider a critical radius r_c that inside it the dynamics is MONDian (i.e., the acceleration is bellow a_0) and outside it the dynamics is Newtonian. The critical radius and corresponding mass scale are

$$r_c = \frac{2a_0}{H_0^2[\Omega_b^0 a^{-3} + 2\Omega_r^0 a^{-4} - 2\Omega_{de}]}, \quad M_c = \frac{a_0}{G} r_c^2. \quad (2)$$

The structures with mass $M > M_c$ are in the Newtonian regime, while for $M < M_c$ are in the MONDian regime. At the beginning of evolution, the dynamics of the structure is Newtonian and the density contrast grows in proportion with scale factor. Afterwards, the structure enters into the MONDian regime and the dynamics significantly deviates from the background expansion (see Fig. 1 of MRH09). In MONDian regime (the scales $M < M_c$), the μ function is approximated as $\mu(x) = x$ and the dynamics of structure is given by

$$\ddot{r} = -\frac{\sqrt{GMa_0}}{r}, \quad (3)$$

where M is the mass and r is the radius of structure. Multiplying the both sides of Eq.(3) in \dot{r} and taking the integration with respect to time, we obtain

$$\dot{r} = \sqrt{v_i^2 - 2(GMa_0)^{\frac{1}{2}} \ln \frac{r}{r_i}}, \quad (4)$$

where r_i is the initial radius at the entrance time into the MONDian regime and v_i is the initial expansion velocity of the structure¹. Changing the time

¹ $v_i = H_i r_i (1 - \delta_i)$ is the peculiar velocity of the structure at the entrance time to MOND regime, where δ_i is the initial density contrast at the entrance time.

derivative to the derivative with respect to scale factor a , Eq.(4) is rewritten as

$$\frac{dr}{da} = \frac{1}{aH(a)} \sqrt{v_i^2 - 2(GMa_0)^{\frac{1}{2}} \ln \frac{r}{r_i}}, \quad (5)$$

where $H(a)$ is given by Eq.(1). The maximum radius of the structure is $r_m = r_i e^\alpha$, where $\alpha = v_i^2 / \sqrt{4GMa_0}$ (Sanders , 1998). Throughout the re-collapse, the global radial velocity converts to the dispersion velocity and eventually the structure virializes at $r_{vir} = r_i e^{\alpha-1/2}$ (for more details see MRH09).

3. Background dynamics and the structure formation in MOND

Here, we calculate the influence of the background dynamics on the MONDian structure formation for two extreme cases of low-density universe and dark energy dominated universe.

3.1. Low-density background

In the low-density background, there is no dark matter and dark energy in the universe and the total density parameter is $\Omega_t^0 = \Omega_b^0 + \Omega_r^0 \sim 0.02$ (Walker et al. , 1991; Carlberg et al., , 1997). We take three class of objects with the masses of $10^6 M_\odot$, $10^{11} M_\odot$ and $10^{13} M_\odot$ which represent the globular cluster, galaxy and cluster of galaxies, respectively. By numerical integration of Eq.(5) the evolution of structures is obtained as a function of redshift. The initial conditions (r_i, v_i) , the maximum radius, r_m , the virialization radius, r_{vir} , together with corresponding redshifts (z_m, z_{vir}) are shown in Table (1). The smaller structures enter the MOND regime and virialize earlier than the larger ones. The dynamical behavior of structure formation in MOND is similar to the hierarchical structure formation in standard CDM model (Padmanabhan , 1993; Malekjani et al. , 2009b). Here we see that the MONDian spherical collapse under a low-density background obtains the virialization at high redshift and therefore can interpret the rapid growth of density perturbations. This result is also discussed in (Sanders , 2001). Sanders assumed two-field Lagrangian-based theory of MOND and obtained the rapid growth of density perturbation by solving the corresponding differential equation in non-linear regime.

3.2. Dark energy background

An interesting theoretical candidate for dark energy is cosmological constant with the time-independent equation of state $\omega = -1$ ² (Zeldovich et al. , 1967; Weinber , 1989; Carroll , 2001). However, the cosmological constant provides excellent fit to the SNIa and CMB data, it suffers from the coincidence and fine tuning problems at the early universe. One solution to these problems is assuming the dynamical dark energy models whose equation of state evolves with cosmic time. In these models an evolving scalar field generates the energy and the pressure of the dark energy, and provides a positive accelerating universe. Here we use the variable dark energy model proposed by Wetterich (2004) in which the equation of state is (Wetterich , 2004)

$$\omega(a; b, \omega_0) = \frac{\omega_0}{[1 - b \ln(a)]^2}, \quad (6)$$

where ω_0 is the state parameter at the present time, a is the scale factor and b is the binding parameter which is related to the amount of dark energy in the universe. However, ω evolves differently for various bending parameters, it asymptotically converges to $\omega_0 = -1$ at the present time (i.e. $a = 1$). In the variable dark energy dominated background, the density parameter of dark energy in Eq.(1) can be described as

$$\Omega_{de}(a; b, \omega_0) = \Omega_{de}^0 a^{-3[1+\overline{\omega}(a; b, \omega_0)]}, \quad \overline{\omega}(a; b, \omega_0) = \frac{\omega_0}{[1 - b \ln(a)]}, \quad (7)$$

where $\Omega_{de}^0 = 8\pi G\rho_{de}^0/3H_0^2$ and $b = 0$ corresponds to the standard cosmological constant model. We assume no clustering for dark energy, but it can influence on the growth rate of the background and consequently on the formation of structures. The various density parameters for variable dark energy model are

$$\Omega_t^0 = \Omega_b^0 + \Omega_{de}^0 = 1.0, \quad \Omega_{de}^0 = 0.98, \quad \Omega_b = 0.02, \quad \Omega_{dm}^0 = 0. \quad (8)$$

Figure (1) shows the evolution of critical length scale r_c , as a function of redshift z , for different cosmological background models calculated from Eq. (2). In the case of low-density model (black solid line), the critical radius increases with the scale factor. It means that the MONDian domains gets larger with time, and therefore the larger structures enter to the

²The equation of state is $p = \omega\rho$

MONDian regime after the smaller ones. The critical radius for the case of standard cosmological constant model (blue-dashed line) evolves same as the low-density model for $z \gg 1$, since the contribution of baryonic matter is dominated compare with cosmological constant in the early times. The cusp in blue-dashed line indicants the equality epoch of baryonic matter and dark energy, hence r_c increases to approach infinity at this time. After the equality epoch, the cosmological constant is dominated and r_c is smaller than that of low-density model. This implies that in the universe dominated by cosmological constant, the structure crosses r_c and enters the MOND regime later than low-density universe. In the case of variable dark energy model with $b = 1$ (red dotted-dashed line), r_c is smaller than other two models at any redshift. For example in the case of $b = 1$, the structure with the mass of $M = 10^{11} M_\odot$ crosses the r_c and enters the MOND regime at $z_{enter} = 68$, while for the low-density model, this happens at $z_{enter} = 146$. It should be noted that, because of the logarithmic term in Eq. (6), the critical radius for all models with $b \neq 0$ converges to the case of $b = 0$ at $z \ll 1$.

In order to obtain the evolution of the structure in MOND under the variable dark energy background, we integrate the Eq.(5) by using Eq.(1). For different values of b , the entrance redshift, the initial radius, and the initial expansion velocity at the entrance time for a galaxy with the mass of $M = 10^{11} M_\odot$ are shown in Table (2). Increasing the bending parameter leads the structure to enter the MOND regime at the lower redshift. Figure (2) shows the evolution of galaxy mass structure for different cosmological background models. The numerical results are summarized in the last four columns of Table (2). The dependence of the virialization redshift and the virialization radius of a galaxy mass structure to the bending parameter are shown in Figure (3). The structure virializes at lower redshift with larger radius for the higher values of bending parameter. The virialization in low-density model is equal to the virialization in variable dark energy dominated background with $b \sim 0.4$.

3.3. Comparison with standard top-hat model

In the framework of standard model of structure formation, the baryonic structures are formed within the potential well of dark matter. One of the simple analytical method for modeling the structure formation is the spherical collapse with top-hat density distribution model (Padmanabhan , 1993; Malekjani et al. , 2009b). In the standard top-hat model, the virialization redshift of the galaxy mass structure takes place at $z_{vir} \sim 0.5$ (see Tab.(1) of

Ref.(Malekjani et al. , 2009b)), while in MOND, in the case of low-density background the virialization takes place at $z_{vir} \sim 17$. As we showed, the variable dark energy background postpones the collapse of structures. According to Table (2), the case of $b = 2$ gives the virialization redshift that is compatible with standard top-hat model.

3.4. Comparison with MONDian N-body simulation

Recently, Linares et al. (2009) have presented cosmological N-body simulation for the large scale structure evolution in the framework of MOND. They studied the effects of the curl-field, $\nabla \times \mathbf{H}$, upon the gravitational structure formation. They also showed that the large scale structure formation is faster in MOND due to strong gravity and found a correlation between the mass of the structure and the formation redshift in Λ CDM and MOND models. This correlation is given by

$$\begin{aligned} \log(M/M_{\odot}) &= -0.8161 \log z_{vir} + 11.5086, & \Lambda CDM \\ \log(M/M_{\odot}) &= -0.8981 \log z_{vir} + 11.4685, & OCBMond \\ \log(M/M_{\odot}) &= -0.6477 \log z_{vir} + 11.4036, & OCBMond2 \end{aligned} \quad (9)$$

where OCBMond denotes to the MOND model without considering curl-field in the solution of the MONDian Poisson's equation whereas OCBMond2 is the solution with the influence of curl-field in the process of structure formation. These linear relations are obtained by the best-fit power laws to the scatter plot (see Fig. 11 of Linares et al. 2009). According to Eq. (9), the evolution of structures in OCBMond2 is slightly faster than in Λ CDM, hence the high-mass objects appear to form at earlier times. There is also a significant difference between OCBMond and OCBMond2 with the former being closer to Λ CDM. This underlines the speed-up impact of the curl-field for the structure formation which is similar to the effect of dark energy. In Figure (4), we plot the mass of structure as a function of virialization redshift, based on Eq.(9). Here, we compare the results of Λ CDM and OCBMond2 simulations with the results of variable dark energy background model. Since the curl field H is neglected in OCBMond simulation and this model can not justified in a fully three dimensional N-body simulation, therefore we do not consider OCBMond simulation in this comparison. In the case of OCBMond2 and Λ CDM the virialization redshift of galaxy mass structure is $z_{vir} \sim 4$ which is comparable with the virialization of galaxy in a variable dark energy background model with $b = 0.97$ (see Fig.4). Here we assume

that the mass of structure does not vary with time after the virialization. Therefore today's mass of the structure is equal with the mass of structure at the virialization time. This comparison can be considered as a technical tool for studying the background fluid of the universe. For example, here we see that the bending parameter b of the dark energy fluid is constrained as $b = 0.97$, using the spherical collapse in MOND.

4. Conclusion

In this paper we dealt with the problem of the structure formation in the framework of Modified Newtonian Dynamics (MOND) under the influence of variable dark energy dominated background model. We utilized the spherical collapse model and showed that the virialization of structures depends strongly on the background cosmology. We showed that the variable dark energy dominated background postpones the virialization of structures. Here the results of three background models including the low-density model, the standard cosmological constant model ($b = 0$), and the variable dark energy model ($b \neq 0$) were compared. In the cosmological constant model, the structures virialize faster than other models (i.e., in the higher redshift). For variable dark energy model, increasing the bending parameter b causes the structure virializes at lower redshift with larger radius. Therefore the variable dark energy model puts off the spherical collapse to the later times. The case of low-density model has an intermediate behavior such that the virialization redshift in this model corresponds with $b = 0.4$ in variable dark energy model. Finally, we compared the virialization of structures under the variable dark energy model with the recent results of MONDian N-body simulations. We showed that the various models of simulation are consistent with the variable dark energy model with different bending parameter.

It should be noted that most of the galaxies are formed within the host environment such as cluster of galaxies and influenced by external gravity. In our model, the structures assumed to be isolated system which is acceptable approximation when the external acceleration field is lower than the critical acceleration. Due to nonlinearity of Poisson's equation in MOND, the strong equivalence principle is violated and consequently the internal dynamics is affected by the external field. Therefore MOND predicts different evolution for structures within the external gravity background. For studying the background dynamics, we assumed the simple baryonic spherical collapse model by avoiding the complexity of neutrino problem which can be appeared in

the scales of clusters of galaxies.

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References

- Bekenstein, J. D., 2004. Phys. Rev. D. 70, 3509.
- Bekenstein, J. D., Milgrom M., 1984. ApJ 286, 7.
- Bosma, A., 1981. AJ, 86, 1825.
- Carlberg, R.G., et al. 1997, preprint, astro-ph/9706115.
- Carroll, S. M., 2001. The Cosmological Constant, Living Rev. Relativity 4, 1.
- Carroll, S. M., *et al.*, 2004. Phys. Rev. D 70, 043528.
- Halle, A., Zhao, H. S., Li, B., 2008. ApJS 177, 1.
- Klypin, A. *et al.*, 1999. ApJ 522, 82.
- Knebe A., *et al.*, 2004. ApJ 603, 7.
- Llinares C., Knebe A., Zhao H., 2008. MNRAS 391, 1778.
- Malekjani, M., Rahvar, S., Haghi, H., 2009. ApJ 694, 1220, (MRH09).
- Malekjani, M., Rahvar, S., Jassur, D. M. Z., 2009. New Astronomy 14, 398.
- Metz M., Kroupa P. & Libeskind N., 2008. ApJ 680, 287.
- Milgrom, M., 1983. ApJ 270, 365.
- Moore, B. *et al.*, 1999. ApJ 524, L19.
- Nusser, A., 2002. MNRAS 331, 909.
- Padmanabhan, T., 1993. Structure Formation in the Universe. Cambridge University Press.

- Pointecouteau, E., & Silk, J. 2005, MNRAS, 364, 654.
- Sanders, R. H., 1998. MNRAS 296, 1009.
- Sanders, R. H., 2001. ApJ 560, 1.
- Sanders 2003, MNRAS, 342, 901 whi88,san03.
- Sobouti, Y., Hasani Zonoozi, A., Haghi, H., 2009. A & A 507, 635.
- Spergel, D. N. *et al.*, 2007. ApJS 170, 377.
- The & White 1988, AJ, 95, 1642.
- van Albada, T. S. *et al.*, 1985. ApJ 295, 305.
- Walker, T.P., Steigman, G., Schramm, D.N., Olive, K.A., Kang, H., 1991. ApJ. 376, 51.
- Weinberg, S., 1989. Rev. Mod. Phys. 61, 1.
- Wetterich C. 2004. Phys. Lett. B 594, 17.
- Zeldovich, Ya. B., Pisma Zh. Eksp. Teor. Fiz., 6, 883 (1967).
- Zhao, H. S., 2007. ApJ 671, L1.

Table 1: Numerical results for the evolution of the structure in the low-density background model for various mass scales in MOND. First column indicates the mass of the structure. Second column is the redshift of entrance time to the MOND regime. The third column is the initial radius of the structure at the entrance time. The forth column is the initial density contrast at the entrance time. Fifth column denotes the initial velocity at the entrance time. r_m and z_m are the maximum radius and corresponding redshift. z_{vir} and r_{vir} are the virialization redshift and the virialization radius of the structure.

M	z_{enter}	$r_i[kpc]$	$\delta_i \times 10^{-5}$	$v_i[km/s]$	z_m	$r_m[kpc]$	z_{vir}	$r_{vir}[kpc]$
$10^6 M_\odot$	848	0.033	1.30	14.5	304	0.079	200	0.047
$10^{11} M_\odot$	147	13.4	7.48	315	27.5	48	17.5	28.4
$10^{13} M_\odot$	74.5	123.7	14.60	1097	8.4	591	4.8	339

Table 2: Numerical results for the evolution of galaxy mass scale ($10^{11} M_\odot$) with the variable dark energy background in MOND. First column indicates the bending parameter of variable dark energy. The second column is the entrance redshift to the MONDian regime. Third column is the initial radius of the structure at the entrance time. Fourth column is the initial velocity of the structure at the entrance time. r_m and z_m are the maximum radius and corresponding redshift. z_{vir} and r_{vir} are the virialization redshift and the virialization radius of the structure.

b	z_{enter}	$r_i[kpc]$	$v_i[km/s]$	z_m	$r_m[kpc]$	z_{vir}	$r_{vir}[kpc]$
0.0	146	13.4	285	40	37.5	27	22.7
1.0	68	29.0	401	6	219.2	4	132.9
1.5	45	43.8	442	2.2	510.8	1.2	309.8
2.0	36	53.7	475	1.0	916.1	0.3	555.6

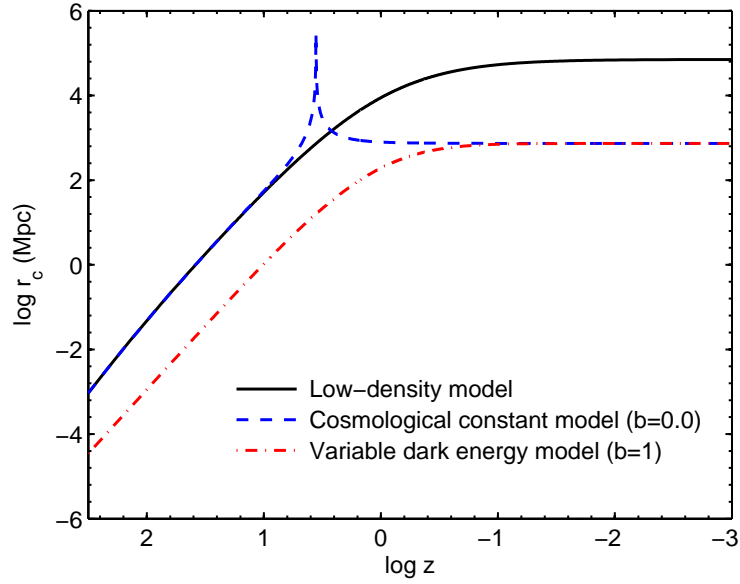


Figure 1: Log-log plot of the MONDian critical radius, r_c , as a function of redshift for various cosmological background models as described in legend. The cusp on the blue dashed line is due to the equality epoch between baryonic matter and dark energy, hence r_c increases to infinity at this time.

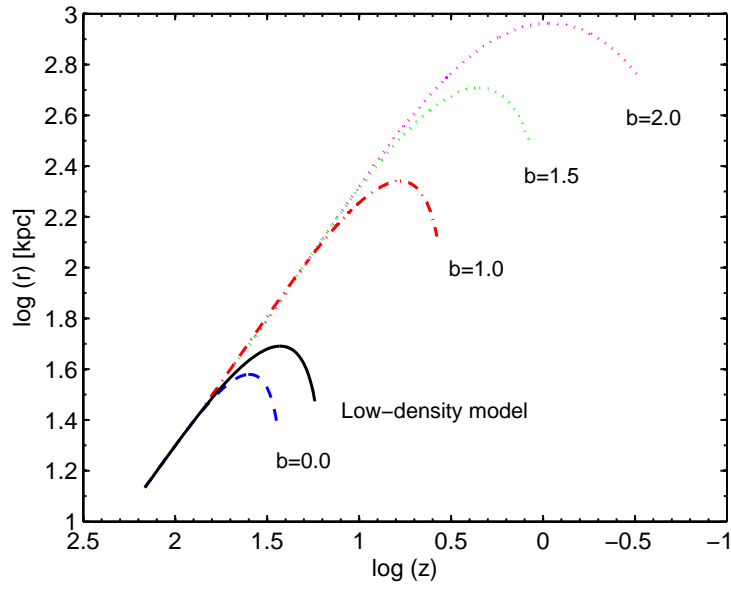


Figure 2: The evolution of a structure with the mass of ($10^{11}M_{\odot}$) from the entrance redshift, z_{enter} until the virialization stage for different values of bending parameter. Increasing the bending parameter causes the slower collapse. The case of $b = 0$ refers to standard cosmological constant background. For comparison, the evolution of the structure in the low-density background is plotted (black solid line).

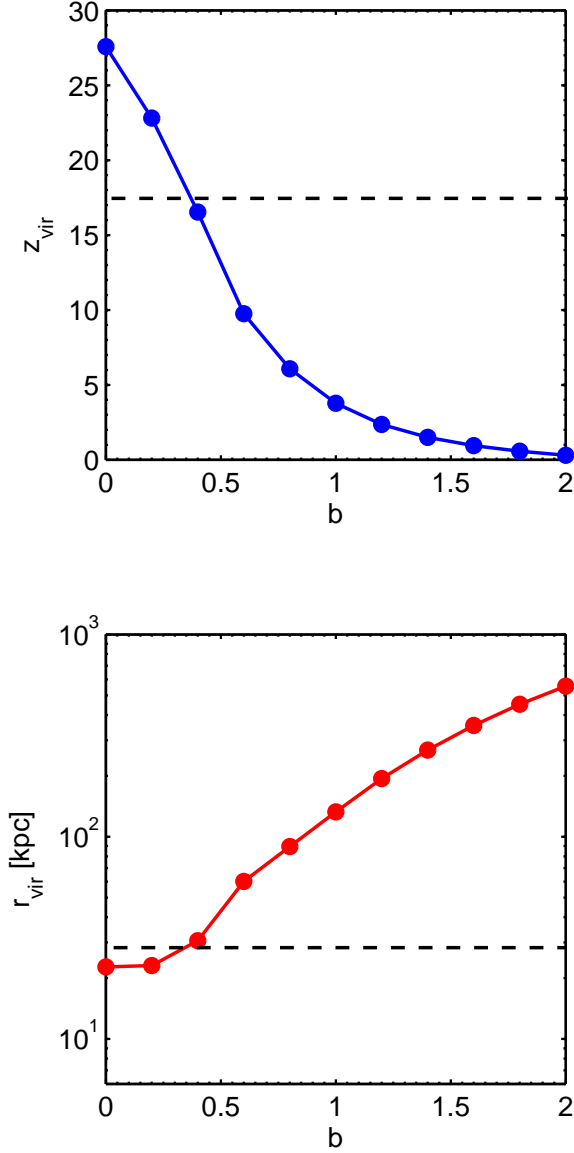


Figure 3: The dependency of the virialization redshift (upper panel) and the virialization radius (lower panel) of $10^{11} M_{\odot}$ structure as a function of bending parameter in variable dark energy dominated background model. By increasing the bending parameter the structure virializes at lower redshift with larger radius. The horizontal dashed line indicates the virialization in low-density background model. The virialization in low-density model is equal with the virialization in variable dark energy background with $b \sim 0.4$.

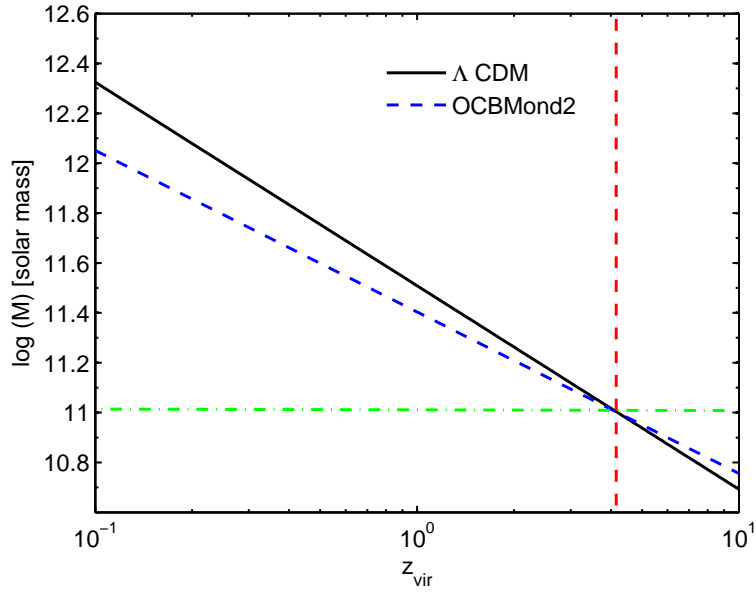


Figure 4: The mass of structure in logarithmic scale as a function of virialization redshift, based on Eq. (9). The horizontal dotted-dashed line represents the galaxy mass structure, the vertical dashed line indicates the virialization redshift of galaxy mass structure in OCBMond2, Λ CDM. The virialization of a galaxy in Λ CDM and OCBMond2 is equal with that of obtained in a variable dark energy model with $b = 0.97$.

Abstract

Keywords:

1.